Robust Adaptive Control for Constrained Dynamical Systems Following Unreliable Reference Signals

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Abstract— **In this paper, we present robust model reference adaptive control laws that guarantee uniform ultimate boundedness of the trajectory tracking error for nonlinear dynamical systems subject to constraints on the state space and the measured output, and affected by matched, unmatched, and parametric uncertainties, as well as uncertainties in the initial conditions. A common assumption in the design of control laws for trajectory tracking is that the reference model or the reference signal verify the constraints imposed on the closed-loop system's trajectory. The control laws presented in this paper do not rely on this assumption and guarantee satisfactory results also in case the reference trajectory or the reference output signal do not verify the given constraints and hence, may draw the plant's trajectory or measured output outside their constraint sets. A numerical example illustrates the feasibility of the theoretical results presented.**

I. Introduction

Robust model reference adaptive control provides a framework to design control laws for nonlinear plants, whose dynamics is partly unknown and affected by matched, unmatched, and parametric uncertainties. Specifically, robust model reference adaptive control laws are able to steer the trajectory of a nonlinear plant toward a given reference trajectory, and guarantee bounded tracking error in finite-time despite external disturbances and modeling errors [1, Ch. 11], [2, Ch. 5].

The presence of constraints on the state space and the measured output, which are imposed by the work environment, the plant's limits of performance, and the output sensors' operative range, add substantial complexity to the problem of designing control systems for linear and nonlinear dynamical systems. The problem of designing control laws for linear plants that account for constraints on the state space has been extensively studied in several publications, such as [3]–[8], to name a few. Additional results on the control of nonlinear plants subject to constraints on the state space and the measured output have been presented in [9] using an invariance control approach, [10], [11] within the context of backstepping control, [12]–[15] applying variable structure controls, [16]–[18] exploiting the model reference adaptive control approach, and [19]–[23] within the framework of receding horizon and model predictive control. To the author's best knowledge, however, existing works rely on the fundamental assumption that the reference signals verify the constraints on the plant state and the measured output.

In this paper, we provide two sets of model reference adaptive control laws that steer the trajectory of nonlinear plants, which are affected by matched, unmatched, and parametric uncertainties, and uncertainties on the plant's initial conditions, so that the closed-loop system's trajectory and the measured output track given reference signals with bounded error. A unique feature of the proposed results is that the closed-loop system verifies constraints on the plant trajectory and the measured output, also in case the reference signal violates these constraints and hence, may draw the closed-loop trajectory and measured output outside their constraint sets. For this reason, the proposed control laws guarantee robustness of the closed-loop system's trajectory also to unreliable or faulty reference signals. Indeed, our results are useful to address control problems, wherein the reference signal is designed incorrectly or ignoring the presence of constraints.

Assuming that the reference signals verify the constraints on the plant state and the measured output, we specialize our framework to provide adaptive control laws, which guarantee that the closed-loop system's trajectory tracking error lays within predefined bounds at all times. These results are advantageous, since classical model reference adaptive control guarantees boundedness of the trajectory tracking error only after a finite-time transient. Moreover, classical model reference adaptive control does not allow to *directly* impose any constraints on the trajectory tracking error. Indeed, classical model reference adaptive control laws allow only to *indirectly* bound the trajectory tracking error by tuning those matrix gains obtained as solutions of the Lyapunov equation.

For brevity, all proofs have been omitted. Interested readers are referred to [24] for detailed proofs, discussions on the results presented, and additional numerical examples.

II. Notation, Definitions, and Mathematical Preliminaries

In this section, we establish notation, definitions, and review some basic results. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of positive real numbers, $\mathbb{R}^n$ the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ the set of $n \times m$ real matrices, and $\mathcal{B}_\varepsilon(x) \subset \mathbb{R}^n$ the *open ball centered at* x *with radius* ε. The *interior* of the set $\mathcal{C} \subset \mathbb{R}^n$ is denoted by $\mathring{\mathcal{C}}$, the *boundary* of $\mathcal{C}$ is denoted by $\partial\mathcal{C}$, and the *closure* of $\mathcal{C}$ is denoted by $\overline{\mathcal{C}}$. The *identity matrix* in $\mathbb{R}^{n\times n}$ is denoted by I_n or I, the *zero* $n \times m$ *matrix* in $\mathbb{R}^{n\times m}$ is denoted by $0_{n\times m}$ or 0, the *transpose* of $B \in \mathbb{R}^{n\times m}$ is denoted by B^{T}, and the *trace* of $A \in \mathbb{R}^{n\times n}$ is denoted by $\mathrm{tr}(A)$. We write $\|\cdot\|$ both for the *Euclidean vector norm* and the corresponding *equi-induced matrix norm*, $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm, and $\mathrm{spec}(A)$ for the *spectrum* of A. The *Fréchet derivative*

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of the continuously differentiable function $V : \mathcal{D} \to \mathbb{R}$ at $x \in \mathcal{D} \subseteq \mathbb{R}^n$ is denoted by $V'(x) \triangleq \frac{\partial V(x)}{\partial x}$.

In order to constrain the adaptation gains, in this paper we employ a Lipschitz continuous form of the projection operator [25], [26]. To define the projection operator, let $\overline{\theta} > 0$ and $\gamma > 0$, and consider the continuously differentiable convex function

$$f_{\rm c}(\theta) \triangleq \frac{(1+\gamma)\|\theta\|^2 - \overline{\theta}^2}{\gamma\overline{\theta}^2}, \qquad \theta \in \mathbb{R}^n. \tag{1}$$

Definition 2.1 ([1, p. 332, 337]): Consider the continuously differentiable function $f_{\rm c}(\cdot)$ given by (1). The *vector projection operator* $\mathrm{proj} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is defined as

$$\mathrm{proj}(\theta, y) \triangleq \begin{cases} y - \frac{\Gamma\left[f_{\rm c}'(\theta)\right]^{\rm T} f_{\rm c}'(\theta)}{\|f_{\rm c}'(\theta)\|_\Gamma^2} y f_{\rm c}(\theta), \\ \qquad \text{if } f_{\rm c}(\theta) > 0 \text{ and } f_{\rm c}'(\theta) y > 0, \\ y, \qquad \text{otherwise,} \end{cases} \tag{2}$$

where $\Gamma \in \mathbb{R}^{n\times n}$ is symmetric and positive-definite and $\|x\|_\Gamma \triangleq \left[x^{\rm T}\Gamma x\right]^{\frac{1}{2}}$, $x \in \mathbb{R}^n$. Given $\Theta = [\theta_1, \ldots, \theta_m] \in \mathbb{R}^{n\times m}$ and $Y = [y_1, \ldots, y_m] \in \mathbb{R}^{n\times m}$, the *matrix projection operator* is defined as

$$\mathrm{Proj}(\Theta, Y) \triangleq \left[\mathrm{proj}(\theta_1, y_1), \ldots, \mathrm{proj}(\theta_m, y_m)\right]. \tag{3}$$

III. Model Reference Adaptive Control and State Constraints

In this section, we address the trajectory tracking problem for nonlinear plants affected by matched and parametric uncertainties, whose dynamics is partly unknown, and whose trajectory must lay in a compact, simply connected constraint set. Specifically, consider the nonlinear time-invariant plant

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\rm T}\Phi(x(t))\right], \qquad x(t_0) = x_0, \qquad t \geq t_0, \tag{4}$$

where $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$, $t \geq t_0$, denotes the *plant's trajectory*, $0 \in \mathcal{D}$, $u(t) \in \mathbb{R}^m$ denotes the *control input*, $A \in \mathbb{R}^{n\times n}$ is *unknown*, $B \in \mathbb{R}^{n\times m}$, $\Lambda \in \mathbb{R}^{m\times m}$ is diagonal, positive-definite, and *unknown*, $\Theta \in \mathbb{R}^{N\times m}$ is *unknown*, and the *regressor vector* $\Phi : \mathbb{R}^n \to \mathbb{R}^N$ is Lipschitz continuous. Both Λ and $\Theta^{\rm T}\Phi(x)$, $x \in \mathcal{D}$, capture the plant's matched and parametric uncertainties, such as malfunctions in the control system, and Λ is such that the pair $(A, B\Lambda)$ is controllable and $\Lambda_{\min} I_m \leq \Lambda$, for some $\Lambda_{\min} > 0$; these assumptions on the plant dynamics (4) can be considered as standard assumptions in model reference adaptive control design [1, p. 282]. Consider also the *reference model*

$$\dot{x}_{\rm ref}(t) = A_{\rm ref}x_{\rm ref}(t) + B_{\rm ref}r(t), \quad x_{\rm ref}(t_0) = x_{\rm ref,0}, \quad t \geq t_0, \tag{5}$$

where $x_{\rm ref}(t) \in \mathcal{D}$, $t \geq t_0$, denotes the *reference trajectory*, $A_{\rm ref} \in \mathbb{R}^{n\times n}$ is Hurwitz, $B_{\rm ref} \in \mathbb{R}^{n\times m}$, and $r(t) \in \mathbb{R}^m$ is piecewise continuous and bounded and denotes the *reference input*. Lastly, let

$$e(t) \triangleq x(t) - x_{\rm ref}(t), \qquad t \geq t_0, \tag{6}$$

denote the *trajectory tracking error* and consider the compact, simply connected *constraint set*

$$\mathcal{C} \triangleq \{x \in \mathcal{D} : h(x) \geq 0\}, \tag{7}$$

where $h : \mathcal{D} \to \mathbb{R}$ is continuously differentiable; the interior of $\mathcal{C}$, which we assume non-empty, is given by $\mathring{\mathcal{C}} = \{x \in \mathcal{D} : h(x) > 0\}$.

The next theorem is the main result of this section and provides an adaptive control law for $u(\cdot)$ so that $x(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$, and eventually tracks with bounded error the reference trajectory $x_{\rm ref}(\cdot)$, also in case $x_{\rm ref}(t) \notin \mathcal{C}$ for some $t \in [t_0, \infty)$. For the statement of this result, let $P \in \mathbb{R}^{n\times n}$ be the symmetric, positive-definite solution of the the Lyapunov equation

$$0 = A_{\rm ref}^{\rm T}P + PA_{\rm ref} + Q, \tag{8}$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric and positive-definite, let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of their arguments, respectively, and let $\overline{x}_{\rm ref,d} \geq 0$ be such that $\|\dot{x}_{\rm ref}(t)\| \leq \overline{x}_{\rm ref,d}$, $t \geq t_0$, where $x_{\rm ref}(\cdot)$ denotes the solution of (5); since $r(\cdot)$ is bounded by assumption, the solution $x_{\rm ref}(\cdot)$ of (5) is bounded [27, p. 245] and hence, $\dot{x}_{\rm ref}(\cdot)$ is bounded. Furthermore, let $\overline{h}_{\rm d} \triangleq \max_{x\in\mathcal{C}} \|h'(x)\|$; since $h'(\cdot)$ is continuous on the compact set $\mathcal{C}$, $\overline{h}_{\rm d}$ is well-defined [27, Th. 2.13]. In addition, let $s(t, x) \in \mathcal{D}$, $(t, x) \in [t_0, \infty) \times \mathcal{D}$, denote the solution of (4) with initial condition x, evaluated at time $t \geq t_0$ [27, p. 71]. Lastly, let

$$\kappa \triangleq \min_{x\in\mathcal{S}_{x_0}} h(x), \tag{9}$$

where $\mathcal{S}_{x_0} \triangleq \{x \in \mathring{\mathcal{C}} : x = s(t, x_0), \text{for some}\, t \geq t_0\}$ denotes the *plant's path*; since $\mathcal{S}_{x_0}$ is compact in $\mathcal{D}$ and $h(\cdot)$ is continuous on $\mathcal{S}_{x_0}$, κ is well-defined [27, Th. 2.13].

Theorem 3.1: Consider the nonlinear plant (4) with $x_0 \in \mathring{\mathcal{C}}$, the constraint set (7), the reference model (5), and the adaptive laws

$$\dot{\hat{K}}_x(t) = -\Gamma_x x(t)\frac{e^{\rm T}(t)P}{h(x(t))}\left[I_n - e(t)\frac{h'(x(t))}{2h(x(t))}\right]B - 2\sigma_x(e(t))\hat{K}_x(t), \quad \hat{K}_x(t_0) = \hat{K}_{x,0}, \quad t \geq t_0, \tag{10}$$

$$\dot{\hat{K}}_r(t) = -\Gamma_r r(t)\frac{e^{\rm T}(t)P}{h(x(t))}\left[I_n - e(t)\frac{h'(x(t))}{2h(x(t))}\right]B - 2\sigma_r(e(t))\hat{K}_r(t), \qquad \hat{K}_r(t_0) = \hat{K}_{r,0}, \tag{11}$$

$$\dot{\hat{\Theta}}(t) = \Gamma_\Theta\Phi(x(t))\frac{e^{\rm T}(t)P}{h(x(t))}\left[I_n - e(t)\frac{h'(x(t))}{2h(x(t))}\right]B - 2\sigma_\Theta(e(t))\hat{K}_\Theta(t), \qquad \hat{\Theta}(t_0) = \hat{\Theta}_0, \tag{12}$$

where $\sigma_x(e) \triangleq \overline{\sigma}_x\|Pe\|$, $e(\cdot)$ is given by (6), $\sigma_r(e) \triangleq \overline{\sigma}_r\|Pe\|$, $\sigma_\Theta(e) \triangleq \overline{\sigma}_\Theta\|Pe\|$, $\overline{\sigma}_x, \overline{\sigma}_r, \overline{\sigma}_\Theta > 0$, the matrices $\Gamma_x \in \mathbb{R}^{n\times n}$, $\Gamma_r \in \mathbb{R}^{m\times m}$, and $\Gamma_\Theta \in \mathbb{R}^{N\times N}$ are symmetric and positive-definite, $\hat{K}_{x,0} \in \mathbb{R}^{n\times m}$, $\hat{K}_{r,0} \in \mathbb{R}^{m\times m}$, $\hat{\Theta}_0 \in \mathbb{R}^{N\times m}$, and P is the symmetric, positive-definite solution of the Lyapunov equation (8). If $x_{\rm ref,0} = x_0$,

$$\frac{\overline{h}_{\rm d}\overline{x}_{\rm ref,d}\lambda_{\max}(P)}{\lambda_{\min}(P)\left|\lambda_{\min}(A_{\rm ref}^{\rm T} + A_{\rm ref})\right|} < \kappa, \tag{13}$$

where κ is given by (9), and there exist $K_x \in \mathbb{R}^{n\times m}$ and $K_r \in \mathbb{R}^{m\times m}$ such that

$$A_{\rm ref} = A + B\Lambda K_x^{\rm T}, \tag{14}$$

$$B_{\rm ref} = B\Lambda K_r^{\rm T}, \tag{15}$$

then the trajectory of (4) with $u = \phi(x, \hat{K}_x, \hat{K}_r, \hat{\Theta})$, where

$$\phi(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta}) = \hat{K}_x^{\mathrm{T}} x + \hat{K}_r^{\mathrm{T}} r(t) - \hat{\Theta}^{\mathrm{T}} \Phi(x) \quad (16)$$

for all $(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta}) \in [t_0, \infty) \times \mathring{\mathcal{C}} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{m\times m} \times \mathbb{R}^{N\times n}$, is such that $x(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$. Moreover, there exist $\varepsilon > 0$ and $\gamma > 0$, and for every $\delta \in (0, \gamma)$, there exists a finite-time $T = T(\delta, \varepsilon) \geq 0$ such that

$$e(t) \in \mathcal{B}_\varepsilon(0), \qquad t \geq t_0 + T. \quad (17)$$

Theorem 3.1 provides sufficient conditions for the plant trajectory $x(\cdot)$ to lay in the interior of the compact connected constraint set $\mathcal{C}$ at all times and track the reference model's trajectory $x_{\mathrm{ref}}(\cdot)$ with uniformly ultimately bounded error [27, Def. 4.4], also in case $x_{\mathrm{ref}}(\cdot)$ does not always lay in $\mathcal{C}$. If the reference trajectory $x_{\mathrm{ref}}(\cdot)$ does not lay in the interior of the constraint set $\mathcal{C}$, then the feedback control law (16) must meet two competing objectives, namely guaranteeing that the trajectory tracking error $e(\cdot)$ is uniformly ultimately bounded and the plant trajectory $x(\cdot)$ lays in the interior of the constraint set $\mathring{\mathcal{C}}$ at all times. To meet these objectives, Theorem 3.1 relies on the conservative assumption that (13) is satisfied. The parameter $\kappa > 0$ defined by (9) can only be estimated, and it can be proven that smaller values of κ imply smaller trajectory tracking error [24]. However, it follows from (13) that arbitrarily small values of κ imply arbitrarily large absolute values of the minimum eigenvalue of $A_{\mathrm{ref}}^{\mathrm{T}} + A_{\mathrm{ref}}$; in this case, the reference model's dynamics should be analyzed using the singular perturbation method [28, Ch. 2].

The next theorem specializes Theorem 3.1 to the case wherein $x_{\mathrm{ref}}(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$, and provides adaptive laws that guarantee uniform asymptotic convergence of the plant trajectory to the reference trajectory.

Theorem 3.2: Consider the constraint set (7), the nonlinear plant (4) with $x_0 \in \mathring{\mathcal{C}}$, the reference model (5), the trajectory tracking error (6), the adaptation laws

$$\dot{\hat{K}}_x(t) = -\Gamma_x x(t) \frac{e^{\mathrm{T}}(t) P}{h(x(t))} \left[I_n - e(t) \frac{h'(x(t))}{2h(x(t))} \right] B,$$
$$\hat{K}_x(t_0) = \hat{K}_{x,0}, \quad t \geq t_0, \quad (18)$$

$$\dot{\hat{K}}_r(t) = -\Gamma_r r(t) \frac{e^{\mathrm{T}}(t) P}{h(x(t))} \left[I_n - e(t) \frac{h'(x(t))}{2h(x(t))} \right] B,$$
$$\hat{K}_r(t_0) = \hat{K}_{r,0}, \quad (19)$$

$$\dot{\hat{\Theta}}(t) = \Gamma_\Theta \Phi(x(t)) \frac{e^{\mathrm{T}}(t) P}{h(x(t))} \left[I_n - e(t) \frac{h'(x(t))}{2h(x(t))} \right] B,$$
$$\hat{\Theta}(t_0) = \hat{\Theta}_0, \quad (20)$$

and the feedback control law $\phi(\cdot, \cdot, \cdot, \cdot)$ given by (16), where $\hat{K}_{x,0} \in \mathbb{R}^{n\times m}$, $\hat{K}_{r,0} \in \mathbb{R}^{m\times m}$, and $\hat{\Theta}_0 \in \mathbb{R}^{N\times m}$. If $x_{\mathrm{ref}}(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$, and there exist $K_x \in \mathbb{R}^{n\times m}$ and $K_r \in \mathbb{R}^{m\times m}$ such that (14) and (15) are verified, then the trajectory of (4) with $u = \phi(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta})$, $(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta}) \in [t_0, \infty) \times \mathring{\mathcal{C}} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{m\times m} \times \mathbb{R}^{N\times n}$, is such that $x(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$, and $\|x(t) - x_{\mathrm{ref}}(t)\| \to 0$ as $t \to \infty$ uniformly in t_0.

Theorem 3.2 proves that if the reference trajectory verifies the constraints on the plant state, then the model reference adaptive control law (16) with adaptive laws (18)–(20) guarantees that the plant trajectory lays in the constraint set at all times and the trajectory tracking error asymptotically converges to zero. In the absence of a constraint set, Theorem 3.2 reduces to the classical model reference adaptive control framework [1, pp. 281-286]. Indeed, if $h(x) = 1$, $x \in \mathcal{D}$, then $\mathcal{C} = \mathcal{D}$ and (18)–(20) reduce to the classical adaptive laws for model reference adaptive control [1, p. 286].

Remark 3.1: Theorem 3.2 can be applied to enforce at all times boundedness of the trajectory tracking error within margins assigned *a priori*.

To appreciate the significance of Remark 3.1, consider, for instance, the constraint set

$$\mathcal{C} = \left\{ x \in \mathcal{D} : \eta - \|x - s_{\mathrm{ref}}(t, x_0)\|_R^2 \geq 0, \right.$$
$$\left. \text{for some } t \geq t_0 \right\}, \quad (21)$$

where $\eta > 0$, $\|x\|_R = \left[x^{\mathrm{T}} R x\right]^{\frac{1}{2}}$, $x \in \mathbb{R}^n$, $R \in \mathbb{R}^{n\times n}$ is symmetric and positive-definite, and $s_{\mathrm{ref}}(t, x)$, $(t, x) \in [t_0, \infty) \times \mathcal{D}$, denotes the solution of (5) with initial condition x at time t [27, p. 71]. In this case, $\mathcal{C}$ is a compact and simply connected subset of $\mathcal{D}$, since the *reference model's path* $\mathcal{S}_{x_0,\mathrm{ref}} \triangleq \{x_{\mathrm{ref}} \in \mathcal{D} : x_{\mathrm{ref}} = s_{\mathrm{ref}}(t, x_0), \text{for some } t \geq t_0\}$ is continuous and bounded, and Theorem 3.2 can be applied to design an adaptive control law so that $x(t) \in \mathring{\mathcal{C}}$, $t \geq t_0$, and hence to impose that $e^{\mathrm{T}}(t) R e(t) < \eta$, $t \geq t_0$, where $e(\cdot)$ denotes the trajectory tracking error.

It is worthwhile to recall that classical model reference adaptive control does not allow to estimate the bounds on the trajectory tracking error (6), since these bounds depend on the plant's unknown parameters [1, pp. 281-285]. Moreover, if the plant dynamics were perfectly known, then the bounds on the trajectory tracking error would be defined by the domain of attraction of the trajectory tracking error dynamics [27, p. 142]. However, in general, domains of attraction can be only estimated [29, Ch. 7], [30]–[32]. Finally, conventional model reference adaptive control only allows to *indirectly* regulate the bounds on the trajectory tracking error by choosing the symmetric positive-definite matrix Q, and hence the symmetric positive-definite matix P, which verify the Lyapunov equation (8). Theorem 3.2, instead, allows to directly impose *any* constraint on the trajectory tracking error that can be modeled as a compact and connected set in the same form as (7).

IV. Constrained Output Tracking

In this section, we show an approach to the trajectory tracking problem in the presence of constraints on the measured output, which is based on the use of the projection operator [25], [26]. Despite the approach presented in Section III, the results presented hereafter allow imposing bounds on the adaptation gains, which are assigned *a priori*. Moreover, the control laws presented in the following guarantee robustness not only to matched and parametric uncertainties, but also unmatched uncertainties. Lastly, considering non-zero initial conditions of the trajectory tracking error dynamics as unmatched disturbances [33, Th. 3.4], the framework presented hereafter allows to account for uncertainties on the plant's initial conditions.

Consider the nonlinear time-varying plant and the plant sensors' dynamics

$$\dot{x}_{\rm p}(t) = A_{\rm p}x_{\rm p}(t) + B_{\rm p}\Lambda\left[u(t) + \Theta^{\rm T}\Phi(x_{\rm p}(t))\right] + \hat{\xi}(t),$$
$$x_{\rm p}(t_0) = x_{\rm p,0}, \qquad t \ge t_0, \quad (22)$$
$$\dot{y}(t) = \theta C_{\rm p}x_{\rm p}(t) - \theta y(t), \qquad y(t_0) = C_{\rm p}x_{\rm p,0}, \qquad (23)$$

where $x_{\rm p}(t) \in \mathcal{D}_{\rm p} \subseteq \mathbb{R}^{n_{\rm p}}$, $t \ge t_0$, denotes the *plant's trajectory*, $0 \in \mathcal{D}_{\rm p}$, $u(t) \in \mathbb{R}^m$ denotes the *control input*, $y(t) \in \mathbb{R}^m$ denotes the *measured output*, $\theta > 0$, $A_{\rm p} \in \mathbb{R}^{n_{\rm p}\times n_{\rm p}}$ is *unknown*, $B_{\rm p} \in \mathbb{R}^{n_{\rm p}\times m}$, $C_{\rm p} \in \mathbb{R}^{m\times n_{\rm p}}$, $\Lambda \in \mathbb{R}^{m\times m}$ is diagonal, positive-definite, and *unknown*, $\Theta \in \mathbb{R}^{N\times m}$ is *unknown*, the *regressor vector* $\Phi : \mathbb{R}^{n_{\rm p}} \to \mathbb{R}^N$ is Lipschitz continuous in its argument, and $\hat{\xi} : [t_0,\infty) \to \mathbb{R}^{n_{\rm p}}$ is continuous in its argument and *unknown*. We assume that $\|\hat{\xi}(t)\| \le \hat{\xi}_{\max}$, $t \ge t_0$, and Λ is such that the pair $(A_{\rm p}, B_{\rm p}\Lambda)$ is controllable and $\Lambda_{\min}I_m \le \Lambda$, for some $\Lambda_{\min} > 0$. The term $\hat{\xi}(\cdot)$ captures the plant's unmatched uncertainties, such as external disturbances. Equation (23) models the plant sensors as linear dynamical systems; this model allows to account for the sensor's transient dynamics, which is characterized by the parameter $\theta > 0$ [34, Ch. 2]. Given the *output reference signal* $y_{\rm cmd} : [t_0,\infty) \to \mathbb{R}^m$, which is continuous with its first derivative, define $y_{\rm cmd,2}(t) \triangleq \dot{y}_{\rm cmd}(t)$, $t \ge t_0$, and assume that both $y_{\rm cmd}(\cdot)$ and $y_{\rm cmd,2}(\cdot)$ are bounded, that is, $\|y_{\rm cmd}(t)\| \le y_{\max,1}$, $t \ge t_0$, and $\|y_{\rm cmd,2}(t)\| \le y_{\max,2}$, where $y_{\max,1}, y_{\max,2} > 0$. Consider also the compact, simply connected constraint set

$$\mathcal{C}_y \triangleq \{y \in \mathbb{R}^m : h_y(y) \ge 0\}, \qquad (24)$$

where $h_y : \mathbb{R}^m \to \mathbb{R}$ is continuously differentiable and $h_y(0) > 0$; we assume that the interior of $\mathcal{C}_y$ is non-empty.

The next theorem is the main result of this section and provides a model reference adaptive control law such that $y(t) \in \mathring{\mathcal{C}}_y$, $t \ge t_0$, and the *output tracking error* $e_y(t) \triangleq y(t) - y_{\rm cmd}(t)$, $t \ge t_0$, is uniformly ultimately bounded, that is, there exist $\varepsilon > 0$ and $\gamma > 0$, and for every $\delta \in (0,\gamma)$, there exists a finite-time $T = T(\delta,\varepsilon) \ge 0$ such that if $e_y(t_0) \in \mathcal{B}_\delta(0)$, then [27, Def. 4.4]

$$e_y(t) \in \mathcal{B}_\varepsilon(0), \qquad t \ge t_0 + T. \qquad (25)$$

This control law is effective also in case $y_{\rm cmd}(t) \notin \mathcal{C}_y$ for some $t \in [t_0,\infty)$. For the statement of this result, let $n \triangleq n_{\rm p} + m$ and $x(t) \triangleq \left[x_{\rm p}^{\rm T}(t), [y(t) - y_{\rm cmd}(t)]^{\rm T}\right]^{\rm T} \in \mathbb{R}^n$, $t \ge t_0$, note that (22) and (23) are equivalent to

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\rm T}\Phi(x_{\rm p}(t))\right] + \xi(t),$$
$$x(t_0) = \begin{bmatrix} x_{\rm p,0} \\ C_{\rm p}x_{\rm p,0} - y_{\rm cmd}(t_0)\end{bmatrix}, \qquad t \ge t_0, \quad (26)$$

where $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$, $\mathcal{D} \triangleq \mathcal{D}_{\rm p} \times \mathbb{R}^m$, $A \triangleq \begin{bmatrix} A_{\rm p} & 0_{n_{\rm p}\times m} \\ \theta C_{\rm p} & -\theta I_m\end{bmatrix}$, $B \triangleq \begin{bmatrix} B_{\rm p} \\ 0_{m\times m}\end{bmatrix}$, $B_1 \triangleq \begin{bmatrix} 0_{n_{\rm p}\times m} \\ -I_m\end{bmatrix}$, and $\xi(t) \triangleq B_1\left[y_{\rm cmd,2}(t) + \theta y_{\rm cmd}(t)\right] + \begin{bmatrix} I_{n_{\rm p}} \\ 0_{m\times n_{\rm p}}\end{bmatrix}\hat{\xi}(t)$, and consider the *reference dynamical model*

$$\dot{x}_{\rm ref}(t) = A_{\rm ref}x_{\rm ref}(t) + B_{\rm ref}y_{\rm cmd}(t),$$
$$x_{\rm ref}(t_0) = x_{\rm ref,0}, \qquad t \ge t_0, \quad (27)$$

where $A_{\rm ref} = \begin{bmatrix} A_{\rm ref,1} & 0_{n_{\rm p}\times m} \\ 0_{m\times n_{\rm p}} & A_{\rm ref,2}\end{bmatrix}$, both $A_{\rm ref,1} \in \mathbb{R}^{n_{\rm p}\times n_{\rm p}}$ and $A_{\rm ref,2} \in \mathbb{R}^{m\times m}$ are Hurwitz, and $B_{\rm ref} \in \mathbb{R}^{n\times m}$. Moreover, let $h(x) = h_y(-B_1x + y_{\rm cmd})$, $(x, y_{\rm cmd}) \in \mathcal{D}\times\mathbb{R}^m$, and

$$\mathcal{C} = \{x \in \mathcal{D} : h(x) \ge 0\}. \qquad (28)$$

Furthermore, let $\overline{x}_{\rm ref,d} \ge 0$ be such that $\|\dot{x}_{\rm ref}(t)\| \le \overline{x}_{\rm ref,d}$, $t \ge t_0$, where $x_{\rm ref}(\cdot)$ denotes the solution of (27), and $\overline{h}_{y,\rm d} \triangleq \max_{y\in\mathcal{C}_y}\|h_y'(y)\|$. Lastly, let $s(t,x) \in \mathcal{D}$, $(t,x) \in [t_0,\infty)\times\mathcal{D}$, denote the solution of (22) with initial condition x, evaluated at time $t \ge t_0$ [27, p. 71]. There exists $\kappa > 0$ such that (9) is verified.

Theorem 4.1: Consider the nonlinear plant given by (22) and (23), the constraint set (24), the augmented dynamical system (26), the reference model (27), and the adaptive laws

$$\dot{\hat{K}}_x(t) = \operatorname{Proj}\left(\hat{K}_x(t), -\Gamma_x x(t)\frac{e^{\rm T}(t)P}{h(x(t))}\cdot\left[I_n - \frac{e(t)h'(x(t))}{2h(x(t))}\right]B\right),$$
$$\hat{K}_x(t_0) = \hat{K}_{x,0}, \qquad t \ge t_0, \quad (29)$$
$$\dot{\hat{K}}_r(t) = \operatorname{Proj}\left(\hat{K}_r(t), -\Gamma_r y_{\rm cmd}(t)\frac{e^{\rm T}(t)P}{h(x(t))}\cdot\left[I_n - \frac{e(t)h'(x(t))}{2h(x(t))}\right]B\right),$$
$$\hat{K}_r(t_0) = \hat{K}_{r,0}, \quad (30)$$
$$\dot{\hat{\Theta}}(t) = \operatorname{Proj}\left(\hat{\Theta}(t), \Gamma_\Theta\Phi(x(t))\frac{e^{\rm T}(t)P}{h(x(t))}\cdot\left[I_n - \frac{e(t)h'(x(t))}{2h(x(t))}\right]B\right),$$
$$\hat{\Theta}(t_0) = \hat{\Theta}_0, \qquad (31)$$

where κ is given by (9), the matrices $\Gamma_x \in \mathbb{R}^{n\times n}$, $\Gamma_r \in \mathbb{R}^{m\times m}$, and $\Gamma_\Theta \in \mathbb{R}^{N\times N}$ are symmetric and positive-definite, $e(t) = x(t) - x_{\rm ref}(t)$, $t \ge t_0$, P is the symmetric positive-definite solution of the Lyapunov equation (8), $\hat{K}_{x,0} \in \mathbb{R}^{n\times m}$, $\hat{K}_{r,0} \in \mathbb{R}^{m\times m}$, $\hat{\Theta}_0 \in \mathbb{R}^{N\times m}$, and $\operatorname{Proj}(\cdot,\cdot)$ is given by (3). If $y(t_0) \in \mathring{\mathcal{C}}_y$ and

$$\frac{\overline{h}_{y,\rm d}\lambda_{\max}(P)\left[\overline{x}_{\rm ref,d} + y_{\max,1}\right]}{\lambda_{\min}(P)\left|\lambda_{\min}(A_{\rm ref}^{\rm T} + A_{\rm ref})\right|} < \kappa \qquad (32)$$

and there exist $K_x \in \mathbb{R}^{n\times m}$ and $K_r \in \mathbb{R}^{m\times m}$ such that (14) and (15) are verified, then the trajectory of (22) and (23) with $u = \phi(t, x, \hat{K}_x, \hat{K}_{\rm cmd}, \hat{\Theta})$, where

$$\phi(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta}) = \hat{K}_x^{\rm T}x + \hat{K}_r^{\rm T}y_{\rm cmd}(t) - \hat{\Theta}^{\rm T}\Phi(x) \quad (33)$$

for all $(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta}) \in [t_0,\infty)\times\mathcal{D}\times\mathbb{R}^{n\times m}\times\mathbb{R}^{m\times m}\times\mathbb{R}^{N\times m}$, is such that $y(t) \in \mathring{\mathcal{C}}_y$, $t \ge t_0$. Furthermore, there exist $\varepsilon > 0$ and $\gamma > 0$, and for every $\delta \in (0,\gamma)$, there exists a finite-time $T = T(\delta,\varepsilon) \ge 0$ such that if $e_y(t_0) \in \mathcal{B}_\delta(0)$, then (25) and (25) is verified.

The next corollary provides sufficient conditions for the output tracking error to verify prescribed bounds. For the statement of this result, consider the compact, simply connected constraint set

$$\mathcal{C}_y = \left\{y \in \mathbb{R}^m : \eta - [y - y_{\rm cmd}(\cdot)]^{\rm T}R\,[y - y_{\rm cmd}(\cdot)] \ge 0\right\}, \quad (34)$$

where $\eta > 0$ and R is symmetric and positive-definite.

Corollary 4.1: Consider the nonlinear plant given by (22) and (23) with $y(t_0) \in \mathring{\mathcal{C}}_y$, the constraint set (34), the augmented dynamical system (26), the reference model (27), and the adaptive laws (29)–(31). If $y_{\rm cmd}(t) \in \mathring{\mathcal{C}}_y$, $t \geq t_0$, and there exist $K_x \in \mathbb{R}^{n\times m}$ and $K_r \in \mathbb{R}^{m\times m}$ such that (14) and (15) are verified, then the measured output of (22) and (23) with $u = \phi(t, x, \hat{K}_x, \hat{K}_r, \hat{\Theta})$, where $\phi(\cdot,\cdot,\cdot,\cdot,\cdot)$ is given by (33), is such that $y(t) \in \mathring{\mathcal{C}}_y$, $t \geq t_0$.

If the measured output reference signal lays in the constraint set (34) at all times, then Corollary 4.1 allows imposing bounds on the output tracking error, which are captured by η and R. This result is similar to Theorem 3.1 of [16].

V. Illustrative Numerical Example

In this section, we illustrate the applicability of the theoretical framework presented in this paper by mean of a numerical example that involves an unstable plant. Specifically, the roll dynamics of a delta-wing aircraft at high angle of attack is captured by [1, pp. 285-291]

$$\begin{bmatrix}\dot\varphi(t)\\ \dot p(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ \theta_1 & \theta_2\end{bmatrix}\begin{bmatrix}\varphi(t)\\ p(t)\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\theta_6\left(u(t)+\theta_6^{-1}\begin{bmatrix}\theta_3\\ \theta_4\\ \theta_5\end{bmatrix}^{\rm T}\begin{bmatrix}|\varphi(t)|p(t)\\ |p(t)|p(t)\\ \varphi^3(t)\end{bmatrix}\right),$$
$$[\varphi(0), p(0)]^{\rm T} = [\varphi_0, p_0]^{\rm T}, \qquad t \geq 0, \quad (35)$$

where $\varphi(t) \in \mathbb{R}$, $t \geq 0$, denotes the roll angle, $p(t) \in \mathbb{R}$ denotes the roll rate, $u(t) \in \mathbb{R}$ denotes the aileron deflection angle, $\theta_1, \ldots, \theta_6 \in \mathbb{R}$ are unknown, and $\theta_6 \neq 0$. Our goal is to design a model reference adaptive control law such that the plant trajectory $x(\cdot)$ lays in the constraint set

$$\mathcal{C} = \left\{x \in \mathbb{R}^n : \eta - x^{\rm T}Rx \geq 0\right\}, \quad (36)$$

where $\eta > 0$ and $R \in \mathbb{R}^{n\times n}$ is symmetric and positive-definite. Moreover, $x(\cdot)$ must track the reference trajectory $x_{\rm ref}(\cdot)$ with bounded error, where $x_{\rm ref}(\cdot)$ denotes the solution of (27) with

$$A_{\rm ref} = \begin{bmatrix}0 & 1\\ -\omega^2 & -2\zeta\omega\end{bmatrix}, \qquad B_{\rm ref} = \begin{bmatrix}0\\ \omega^2\end{bmatrix}, \quad (37)$$

$\zeta \in (0,1)$, $\omega > 0$, and $r(t) = \sin t$, $t \geq 0$. To meet these design goals, we apply Theorem 3.1.

The nonlinear plant (35) is in the same form as (4) with $n = 2$, $m = 1$, $N = 3$, $\mathcal{D} = \mathbb{R}^n$, $x = [\varphi, p]^{\rm T}$, $A = \begin{bmatrix}0 & 1\\ \theta_1 & \theta_2\end{bmatrix}$, $B = [0,1]^{\rm T}$, $\Lambda = \theta_6$, $\Theta = \theta_6^{-1}[\theta_3,\theta_4,\theta_5]^{\rm T}$, $\Phi(x) = [|\varphi|p, |p|p, \varphi^3]^{\rm T}$, $x_0 = [\varphi_0, p_0]^{\rm T}$, and $t \geq t_0$. In this case, one can verify that the pair $(A, B\Lambda)$ is controllable if and only if $\theta_6 \neq 0$. Moreover, the constraint set (36) is in the same form as (7) with $h(x) = \eta - x^{\rm T}Rx$, $x \in \mathbb{R}^n$, the matching conditions (14) and (15) are verified by

$$K_x = -\theta_6^{-1}\begin{bmatrix}\omega^2+\theta_1\\ 2\zeta\omega+\theta_2\end{bmatrix}, \qquad K_r = \theta_6^{-1}\omega^2, \quad (38)$$

and $\overline{h}_{\rm d} = \max_{x\in\mathcal{C}}\|h'(x)\| = 2\|Rx\|$, $x \in \partial\mathcal{C} = \{x \in \mathbb{R}^2 : x^{\rm T}Rx = \eta\}$. Thus, if (13) is verified, then it follows from Theorem 3.1 that the adaptive control law (16) guarantees that $x(t) \in \mathring{\mathcal{C}}$, $t \geq 0$, and the trajectory tracking error is bounded, that is, (17) is verified with $t_0 = 0$ and for some finite-time $T \geq 0$.

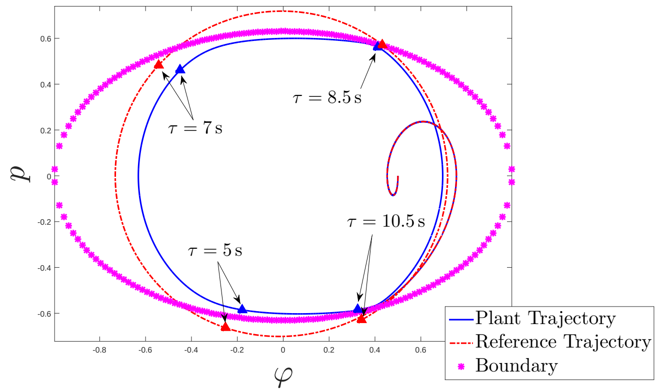

Fig. 1. Closed-loop system's trajectory.

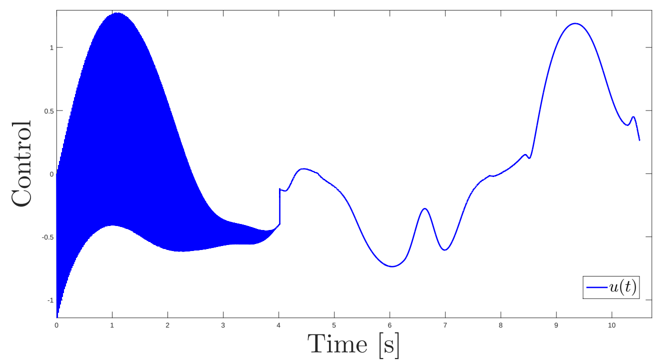

Fig. 2. Closed-loop system's trajectory.

Let $\theta_1 = -0.018$, $\theta_2 = 0.015$, $\theta_3 = -0.062$, $\theta_4 = 0.009$, $\theta_5 = 0.021$, $\theta_6 = 0.75$, $\zeta = 0.7$, $\omega = 1$, $R = \begin{bmatrix}0.4 & 0\\ 0 & 1\end{bmatrix}$, $\eta = 0.3980$, $\Gamma_x = 500I_2$, $\Gamma_r = 500$, $\Gamma_\Theta = 500I_3$, and $Q = \begin{bmatrix}1 & 0\\ 0 & 10^3\end{bmatrix}$. Figure 1 shows the solution $x(\cdot)$ of (35) with control law (16), the solution $x_{\rm ref}(\cdot)$ of (5), and the boundary of the constraint set $\partial\mathcal{C}$. Figure 1 also shows both $x(\tau)$ and $x_{\rm ref}(\tau)$ for $\tau \in \{5, 7, 8.5, 10.5\}\,$s. Clearly, the plant trajectory is always contained in the interior of the constraint set $\mathcal{C}$ and tracks with bounded error the reference trajectory, also in case $x_{\rm ref}(\tau) \notin \mathring{\mathcal{C}}$, such as for $\tau = 5\,$s, $\tau = 8.5\,$s, and $\tau = 10.5\,$s. Figure 2 shows the control input $u(\cdot)$. The high values of the gains Γ_x, Γ_r, and Γ_Θ guarantee highly satisfactory trajectory tracking and induce high-frequency oscillations in the feedback control law (16) for $t \in [0, 4.18)\,$s; at $t = 4.18\,$s, the reference trajectory leaves the constraint set $\mathcal{C}$ for the first time. For practical implementations, lower values of the adaptive gains can be employed to avoid these high-frequency oscillations. On the other hand, lower values of the adaptive gains lead to larger trajectory tracking errors, while the reference trajectory is outside the constraint set.

VI. CONCLUSION

In this paper, we provided model reference adaptive control laws for constrained nonlinear dynamical systems that guarantee robustness to uncertainties both in the plant model and the reference signal tracked, which is supposed to be tracked by the closed-loop trajectory or the measured output. Specifically, given a nonlinear plant, whose dynamics is affected by matched and parametric uncertainties, we provided a control law, which guarantees that the closed-loop system's trajectory traks some reference signal with bounded error and is constrained to some bounded compact set at all times. We also provided a control law, which guarantees that the closed-loop system's measured output traks some reference signal with bounded error and is constrained to some bounded compact set at all times, despite matched, unmatched, and parametric uncertainties, and uncertainties in the plant's initial conditions. The proposed control laws guarantee that the trajectory tracking error is uniformly ultimately bounded and the constraints on the closed-loop system's dynamics are verified also in case the reference signals do *not* verify these constraints and hence, may draw the plant trajectory or the measured output outside their constraint sets. To the author's best knowledge, existing control laws that guarantee satisfactory trajectory tracking for constrained dynamical systems presuppose that the reference signals verify the given constraints.

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